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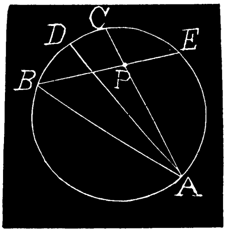
32. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Find the average area of the random sector whose vertex is a random point in a given circle.

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.



Let P be the random point. Through P draw the chords AC, DE forming the sector DPC. From A draw the diameter AB and the chord AD. Let $AB=2r$, $AP=z$, $\angle BAP=\varphi$, $\angle BAD=\theta$, area $DPC=u$, Δ=required average. Then

$$u=r^2(\varphi-\theta-\tfrac{1}{2}\sin2\theta+\tfrac{1}{2}\sin2\varphi)-rz\cos\theta\sin(\varphi-\theta).$$

The limits of θ are $-\frac{1}{2}\pi$ and $+\frac{1}{2}\pi$; of φ, θ and $\frac{1}{2}\pi$; of z, 0 and $2r\cos\varphi=z'$.

$$\therefore\ \Delta=\frac{\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\int_{\theta}^{\frac{1}{2}\pi}\int_{0}^{z'}ud\theta d\varphi zdz}{\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\int_{\theta}^{\frac{1}{2}\pi}\int_{0}^{z'}d\theta d\varphi zdz}=\frac{2}{\pi^2r^2}\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\int_{\theta}^{\frac{1}{2}\pi}\int_{0}^{z'}ud\theta d\varphi zdz,$$

$$=\frac{2r^2}{3\pi^2}\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\int_{\theta}^{\frac{1}{2}\pi}[3\cos^2\varphi(2\varphi-2\theta-\sin2\theta+\sin2\varphi)-8\cos\theta\cos^3\varphi\sin(\varphi-\theta)]d\theta d\varphi$$

$$=\frac{r^2}{12\pi^2}\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}(3\pi^2-12\pi\theta+\theta^2-16\cos^2\theta+4\sin^2\theta\cos^2\theta)d\theta=\frac{37\pi r^2}{144}-\frac{5r^2}{8\pi}.$$

Also solved by the *PROPOSER*.

33. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Find the average area of all regular polygons having a constant apothem.

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let a=constant apothem, $2x$=side, 2θ=central angle of polygon.

$$\therefore\ \frac{\pi}{\theta}=\text{number of sides},\ \frac{\pi ax}{\theta}=\text{area of polygon}.$$

$$\therefore\ A=\text{average area}=\pi a\frac{\int_0^{a\sqrt{3}}\frac{xdx}{\theta}}{\int_0^{a\sqrt{3}}dx}=\frac{\pi}{\sqrt{3}}\int_0^{a\sqrt{3}}\frac{xdx}{\theta}$$

$$=\frac{\pi a^2}{\sqrt{3}}\int_0^{\frac{1}{3}\pi}\frac{\tan\theta\sec^2\theta d\theta}{\theta}\ \text{where } x=a\tan\theta$$